# Failure of the Isothermal Atmospheric Model in Study of Jupiter's Atmosphere: Revised Model Proposed and Tested

Andrew Valentini
*Carthage College, Kenohsa, Wisconsin*
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The development of accurate atmospheric models is crucial to the study of planets, which is directly applicable to the safety and success of spaceflight missions. In this paper, we demonstrate the failure of an isothermal atmospheric model in predicting the behavior of Jupiter's atmosphere, as captured by the Galileo probe's available data as it fell through the planet's atmosphere. We propose a revision to this model and show that this results in a much more accurate model for the planet's atmospheric behavior. Discussion on the minimization of error between this revised model and the data is also provided.

## I. INTRODUCTION

By the application of basic thermodynamics, there exist models that allow us to describe generalized atmospheric conditions subject to assumptions and applicable under specific conditions. Due to their generality, these models should allow the atmosphere of any planet to be described, meaning they enable us to answer hypothetical questions about what life on these planets would be like. We use data collected by the Galileo probe which, launched in 1989, was later given the terminal mission in 2003 of being sent into the interior of Jupiter and collecting relevant atmospheric data during its descent [2]. Since the Galileo probe was sent into the atmosphere of Jupiter, its data provides the opportunity to analyze how well current models describe its physical conditions. In this paper, we focus on Jupiter's dynamic pressure and temperature behavior with respect to altitude, the former of which is termed the environmental *lapse rate* and the latter initially being described by an *isothermal* assumption. In the following section, we derive models to describe this behavior from principles motivated by thermodynamics.

## II. THEORETICAL MODELS

In considering the force balance present in a slab of material held at a constant temperature within an atmosphere, one finds the variation of pressure with respect to altitude experienced by this slab to be of the form:

$$\frac{dP}{dz} = -\frac{mg}{kT}P \tag{1}$$

Where $m$ is the average mass of the molecules in the atmosphere, $k$ is Boltzmann's constant, and $g$ is the gravitational constant of the planet considered. This separable differential equation can be solved for $P$ to arrive at an isothermal expression for how pressure changes with altitude (since there is no change in temperature considered):

$$P = P_0 e^{-mgz/kT} \tag{2}$$

With this equation, a useful term can now be defined, called the *atmospheric scale height* $h$, which is the height by which you would need to raise an observer such that the atmospheric pressure would change by a factor of $e$, where $h$ is given as $h = kT/mg$.

If the adiabatic behavior of an atmosphere is also considered, we can also construct a model for how the temperature of an atmosphere should change with pressure. This is done by starting with the definition of an adiabatic gas:

$$V_f T_f^{f/2} = V_i T_i^{f/2} \tag{3}$$

The ideal gas law can be applied to find an expression for $dT/dP$, which is of the form:

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P} \tag{4}$$

Where $f$ is the average number of degrees of freedom in the atmospheric gas considered. The chain rule for derivatives is then employed on equations (1) and (3) to find an isothermal expression for how the temperature of an atmosphere changes with altitude ($dT/dz$):

$$\frac{dT}{dz} = -\frac{2}{f+2}\frac{mg}{k} \tag{5}$$

Since the right side of this expression is just a constant, we denote this by $\Gamma$ and define it to be the *dry adiabatic lapse rate*:

$$\frac{dT}{dz} = -\Gamma \tag{6}$$

None of the previous equations are restricted to Earth's atmosphere and can be generally applied to any planetary atmosphere with a gravitational constant, $g$. To tailor these equations to the atmosphere of Jupiter, the only necessary modification of these models is thus to revise $g$ to reflect the gravitational constant on Jupiter. To do this, consider the gravitational force experienced by any test mass $m_t$ in radial distance $r$ from Jupiter and arrive at the expression:

$$m_t g = \frac{GM_J m_t}{r^2} \implies g = \frac{GM_J}{r^2} \tag{7}$$

This expression for $g$ can then be substituted into our equation for the dry adiabatic lapse rate (eqaution (5)) and our isothermal atmospheric model (equation (2)) to be tailored to Jupiter.

Even with this revision, the reader may be reluctant to believe that the isothermal atmospheric model accurately describes how the pressure of Jupiter's atmosphere changes with respect to altitude since it assumes that temperature is a constant value. We expect that the atmospheric temperature would surely change as altitude does, so we propose the following revision to the temperature variable in equation (2):

$$T(z) = T_0 + \Gamma z \tag{8}$$

This revision is motivated by the fact that eqaution (5) is a separable differential equation, allowing us to arrive at an expression for how temperature changes with respect to altitude.

Updating equation (1) with this revision for the temperature dependence, we get:

$$\frac{dP}{dz} = -\frac{mg}{k(T_0 + \Gamma z)} \tag{9}$$

Equation (8) can be revised to the following form:

$$\frac{dP}{dz} = -A\frac{1}{1+bz}P \tag{10}$$

if we let $A = \frac{(f+2)\Gamma}{T_0}$ and $b = \frac{\Gamma}{T_0}$. Again, since this is a separable differential equation, we may solve for $P$ as a function of z, which results in the revised model for atmospheric pressure as a function of altitude:

$$P = P_0 \left(\frac{1+bz}{1+bz_0}\right)^{-A/b} \tag{11}$$

Since we expect that this model more accurately describes an atmosphere's pressure at a given altitude, it is the model that is expected to provide the best fit to the Galileo probe data in the following section.

## III. DATA ANALYSIS

Since we know that the change in atmospheric temperature with respect to altitude depends on a constant, $\Gamma$, called the dry adiabatic lapse rate (DALR) from equation (5), we can check the this equation's validity by numerically solving for it given the relevant constants of Jupiter and comparing it to the value derived from fitting a linear line to the temperature and altitude data. To numerically solve for $\Gamma$, we need values for the average molecular mass and the average degrees of freedom of the gas in Jupiter's atmosphere. Assuming that the atmosphere of Jupiter is only comprised of HE and H2, we sum the average molecular mass $m \approx 3.32 \times 10^{-27}$ kg (the average mass of particles in the atmosphere) by assuming that Jupiter's atmosphere is only comprised of HE and H2 and taking their respective concentrations into account. We also infer that $f \approx 4.72$ by the assumption that the atmosphere is only made up of HE and H2 so so the average value for the atmosphere's degrees of freedom can be found by taking the average of the two with, again, consideration for the concentration of each element in Jupiter's atmosphere. With these values, we use equation (5) to conclude that $\Gamma \approx -1.8$ K/km. Using a linear fit between the collected data's altitude and temperature, we find that the experimental value of $\Gamma$ is $-1.97$ C/km:

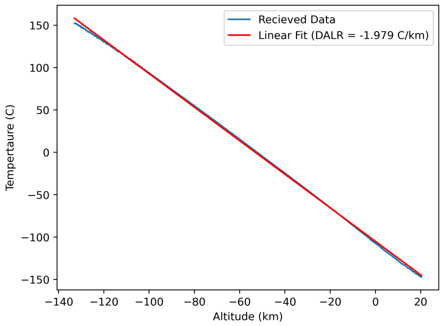

Figure 1. Inferring the $\Gamma$ value of Jupiter by curve fitting the collected temperature and altitude data from the Galileo probe.

Since the theoretical dry adiabatic lapse rate, $\Gamma$, is almost exactly the value inferred from the data, we are able to conclude that Jupiter's atmospheric temperature is linearly proportional to its altitude, meaning that the Galileo probe did not encounter any unexpected atmospheric conditions, such as convection currents. We can also now deduce the scale height value $h$ for Jupiter given what we know about its physical constants and the expression for $h$ (coming from equation (2)). We again assume that $m$, the average molecular mass of Jupiter's atmosphere, is found by taking the average molecular mass of HE and H2 with consideration of their concentration, and we take $T$, the average temperature of Jupiter's atmosphere, to be 163° K, as found in [1]. This results in a scale height value of 38584 meters, meaning we predict Jupiter's atmosphere to change by 1 degree Kelvin for every 38583 meters.

With equation (2), we are able to model Jupiter's atmospheric pressure as a function of altitude. When we compare this model against the data, however, we find a relatively large discrepancy between the two:

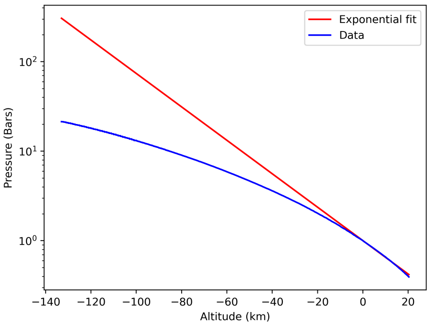

Figure 2. This figure shows the fit between our isothermal atmospheric model and the recieved pressure versus altitude data. The fit is clearly insufficent to capture the complete behavior of the data.

Since equation (2) includes a constant term, $P_0$, we are then free to use various values for this. To show our model's dependence on this term, we plot three models with atmospheric pressure and altitude data, one with $P_0$ as the first pressure value taken by the Galileo probe, the last pressure data taken, and the pressure of one atmosphere. Since these pressure values also correspond to varying altitude and temperature, these parameters were also selected at the varied pressure values. This is compared against the data to emphasize the failure of the isothermal models:

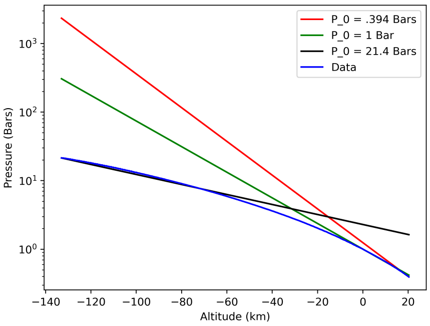

Figure 3. Our isothermal atmospheric model with varied initial pressure values to emphasize the failure of this model.

The fit lines in this figure and the previous are tangent to the data at varying points because when we fix an initial pressure value, this also fixes an initial altitude and temperature value. These initial values contribute to the corresponding fit lines being tangent to the data curve but, since the behavior is still governed by the isothermal atmospheric model, this agreement between our fits and data does not extend beyond the point where our fits are tangent.

As explained in the previous section, we make an alteration to the isothermal atmospheric model by revising the assumption that the temperature is constant. With the revised model given by equation (11), we can now make a more precise fit to the pressure versus altitude data:

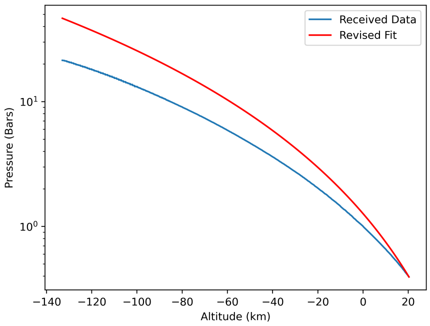

Figure 4. Our revised model fitted against the data. A much better fit is shown to be made than the original isothermal models in figure 3.

This model provides a much better fit to the data than our original isothermal atmospheric model (Figure 2) which, again, is to be expected since the atmosphere of Jupiter varies in temperature with altitude. The fit line in the figure above was created with initial conditions at the first data point taken by the probe, which is why the fit is very strong near the right side of the plot with a quick decay at pressure decreases. To quantify our model's fit to the data, we calculate the difference between the two curves at every altitude:

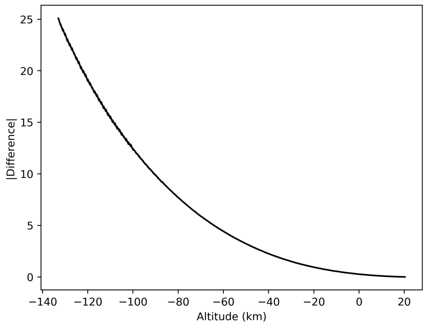

Figure 5. The difference between our model and the pressure versus altitude data for Figure 4.

Since equation (11) carries an independent parameter, $z_0$, that is treated as an initial condition, our model can be tested against the data at multiple values for this parameter to determine the most accurate fit. To do this, we loop over all the altitude values the Galileo probe provides, use each as the $z_0$ parameter, produce our model with these values and calculate the difference between each of these models and the real data. We find that the model that minimizes the discrepancy between our model and the data is $z_0 = 9.12$ km which results in a total error of 270. For reference, the total error produced by figure 4 was 4107. To further reduce error, the additional free parameters in equation (11), $P_0$ and $T_0$,could be selectively chosen in a similar fashion to find the combination of parameters that leads to the most accurate fit.

## IV. CONCLUSIONS

It has been shown that an isothermal atmospheric model is insufficient in modeling an physical atmosphere since we expect all atmospheres to vary in temperature with altitude. To arrive at a more predictive model, we revised the constant temperature assumption in the isothermal model. This revision is shown to fit the data received by the Galileo probe as it fell through the atmosphere of Jupiter and we provide a method by which the error in this revised model may be reduced to more accurately fit the data. Future work could revise our proposed model by identifying any other parameter dependencies, which would likely result an even better prediction for the atmosphere of Jupiter. Extremely well-predictive models of these kind are crucial for future space missions that involve the consideration of a planet's atmosphere.

## REFERENCES

[1] NASA Solar System Exploration. "Solar System Temperatures". `https://solarsystem.nasa.gov/resources/681/solar-system-temperatures/`. 2022.

[2] A.P. Showman. "Jupiter and the Outer Planets". In: *Planetary Atmospheres* (2003), pp. 1730–1745.