Thermodynamics of Rankine Cycles: Analysis of Modern Power Plants

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The Rankine cycle is a lynchpin of modern society; providing a very efficient engine for the production of usable energy. Today, one of the most important implementations of the Rankine cycle is in various power plants. This paper first details the basic thermodynamics present in the Rankine cycle with extensive discussion of the phase transition required by it before discussing the application of this cycle to three forms of power plants used today. These power plants are then described by their efficiencies and are analyzed by considering the various forms of energy created during each step in the Rankine cycle. I then suggest avenues for extensions of this work and how the Rankine cycle could be made more efficient.

I. INTRODUCTION

Due to the efficiency and reliability of the Rankine cycle, this engine is largely responsible for our modern industrial world. The Rankine cycle is a steam engine that heats liquid water into energetic steam, causing the volume that holds this liquid to expand, thereby doing work on the environment. This work created by the Rankine cycle is used by the steam engine in a turbine, which creates a source of usable energy. In this study, we analyze a implementation of the Rankine cycle which is crucial to our modern existence. The majority of the power plants that support our civilization make use of the Rankine cycle, which allows us to use the thermodynamic tools known about these engines to understand how these power plants operate and to analyze their efficiency. We consider three types of power plants that make use of the Rankine cycle; a nuclear power plant, a typical fossil fuel model, and a supercritical turbine. Information on the operating temperatures, pressures, and other specs of these power plants are taken from [2], [3], [1] and data for the properties of water under the liquid-to-gas phase transition which will be used in this study are provided by [4].

II. WATER PHASE DIAGRAM

It should first be stated that the the Rankine cycle is composed of the following four stages: an isothermal phase where water is pumped into the power plant's engine, an isobaric heating of this water until it fully becomes steam, an isentropic expansion of this steam that powers the turbine of the engine, and a final stage where the steam is condensed back into water. To analyze this implementation of the Rankine cycle, we first consider how the Gibbs free energy and entropy change during each phase of the Rankine cycle since these will reveal the phase transition between liquid and gaseous water that the Rankine cycle requires.

Since the Gibbs free energy is only defined in terms of extensive variables (it is an NPT ensemble), displaying its plot over the duration of the cycle will reveal the phase transition that occurs during the heating and condensing stage of the Rankine cycle. The change in Gibbs free energy is defined as:

$$dG = VdP - SdT + \mu dN \tag{1}$$

Since we are not changing the number of particles in this system, $dN = 0$. This leaves us with the expression $dG = VdP - SdT$. Because the entropy for liquid water is different than the entropy for gaseous water, there will be a discontinuity in the plot of Gibbs free energy at the region of the phase transition when plotted with respect to pressure and temperature. This is shown in the following plot:

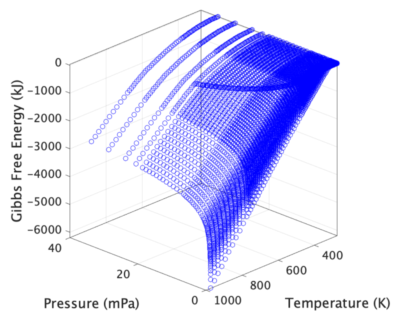

Figure 1. The phase transition between liquid and gaseous water during the Rankine cycle is shown by the difference in slopes of the two planar regions near the dark line across the plot.

Fig. (1) shows that when the entropy, S, instantaneously changes at the phase transition, there must be a difference in the slope of the Gibbs free energy plot between the two regions due to the dependence on S in Eq. (1). This figure also shows where the critical point for water occurs. The critical point is the point at which the water, when supplied with enough energy, no longer undergoes a phase transition between a liquid and gaseous state and becomes a supercritical fluid.

Because the water does not undergo a phase transition in the supercritical region, which has been previously explained in this paper to exhibit a discontinuity in the Gibbs free energy plot, this supercritical point is identified where the discontinuity indicating the phase transition in Fig. (1) ceases to exist. Because there is no longer a phase transition beyond the supercritical point, the slope of the plane in the previous plot would not be expected to exhibit the sudden discontinuity, which is what is observed in the plot.

This phase transition can also be observed in a plot comparing entropy and temperature.

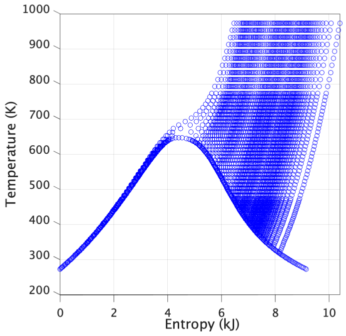

Figure 2. Temperature of the water plotted with respect to its entropy. The gap in this diagram represents the region where the water is in an intermediary mixture of both liquid and gas during the heating phase of the Rankine cycle.

Because the data provided by [4] that produced Fig. (2) only considers water at a single state; water that is either in a fully liquid or gaseous state, the large white gap in the plot underneath the inverted parabola represents the region where the water has become a mixture of both liquid and gas. This allows us to conclude that a homogeneous substance cannot exist in the gap shown above since the plot does not have any data points inside of the inverted parabola. Because this gap represents a region where only a mixture between liquid and gaseous water can exist, this implies that only a mixture of phases can exist inside of this gap, which would require data that is not provided by [4].

From the analyses detailed in this section, it was surprising to me that there is a point beyond which phase transitions stop happening. I didn't know that a supercritical region where both liquid and gaseous water could simultaneously exist. This study also forced me to think about the nature of the PV and TS diagrams and how they are related through a higher-dimensional plot. I was not thinking of this connection before this project but this project has brought it forth, which allows for a better mental picture for the underlying process.

III. RANKINE CYCLES

Now that we have analyzed the water phase diagram for the Rankine cycle and have detailed the phase transition that occurs during the engine's operation, we can begin considering the engine's operation itself and how it is used in power plants today. To do this, we must first determine the operating conditions of each of the real power plants considered in this study. From [2], we find that the operating pressure for the nuclear power plant is 7.5 MPa and its operating temperature is 290 degrees Celsius. From [3], we find that the operating pressure for the traditional fossil fuel-based power plant is 28 MPa and its temperature is 600 degrees Celsius. Finally, from [1], we find that the operating pressure of the supercritical power plant is 33 MPa and its operating temperature is 650 degrees Celsius. We consider all of these power plants to operate with an initial temperature around 20 degrees Celsius, which should be noted is near room temperature.

For the nuclear power plant, we generate the following pressure-volume diagram with the red line indicating each of the steps made by the steam engine in its operation.

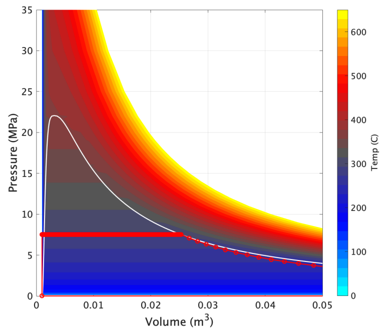

Figure 3. The pressure-volume diagram generated for the nuclear power plant. The red lines indicate the steps taken by this process and the white line represents the phase boundary between the liquid and gaseous water. The region inside of the white line is a mixture of liquid and gaseous water with the region to the right of the white line representing a state of pure steam with the small region to the left of the white line representing a state of pure liquid water.

Due to the first law, $U = Q + W$, the heat required to operate this Rankine cycle, Q_h, can be determined from this plot by subtracting the area under the isobaric heating line (the boiling phase of the steam engine which causes the container to expand with a constant pressure) from the change of internal energy of the water during this process. The work that is generated by the Rankine cycle which moves the turbine of the engine to create usable energy, can be determined from this plot by just considering the area under the isentropic red line that borders the white phase boundary.

The Carnot efficiency; the *ideal* efficiency of an engine operating between the given minimum and maximum temperature, is mathematically expressed as:

$$e \leq 1 - \frac{T_c}{T_h} \tag{2}$$

where we take T_c to be the initial temperature of the power plant (20 degrees Celsius) and T_h to be the operating temperature of each of the three power plants considered in this study. It should be noted that the idealized engine is one in which $T_h >> T_c$, making $e = 1$ and the non-ideal engine is the one in which $T_c = T_h$, making $e = 0$, which means that a more efficient engine is the one whose efficiency is closer to 1. Using Eq. (2), we find that the Carnot efficiency of the nuclear power plant is .479, .664 for the traditional fossil fuel power plant, and .682 for the supercritical turbine.

The engine efficiency, defined to be the ratio of the steam engine's output of work to the heat required to operate the engine ($e = W/Q_h$), can be calculated for each of the three power plants considered where, again, the work produced by the engine is the area under the isentropic red line in Fig. (3) and the heat used to operate the engine is the difference between the change in internal energy of the water during the heating phase and the area underneath the isobaric red line in Fig. (3). It should be noted that Q_h could be more easily obtained by only considering the area under this isobaric red line in the temperature-entropy space. We find that the engine efficiency is .364 for the nuclear power plant, .410 for the traditional fossil fuel power plant, and .418 for the supercritical power plant. From these calculations, we find that the Rankine cycle becomes increasingly more efficient as the operating temperature and pressure of the power plant increase for the power plant since the supercritical power plant displays both the highest engine and Carnot efficiency and has the highest operating pressure and temperature out of the three power plants considered. It should be noted that all of the engine efficiency values should be expects to be lower than the corresponding Carnot efficiency values since the Carnot efficiency is defined as the maximum possible efficiency of an engine that operates between two given temperatures where the engine efficiency is the efficiency that is physically observed for each of the power plants.

Another set of tools we have at our disposal to analyze these power plants is to consider how the internal energy, enthalpy, and Gibbs free energy change with each step in the cycle. Because enthalpy is the energy considered when volume can be freely exchanged between a reservoir and environment, plotting enthalpy at each step in the steam engine cycle allows you to analyze how the energy of the power plant at each step in its cycle either corresponds to heat being added to it or work being done on the power plant's environment (which is what creates the usable energy) without having to consider the effect of the phase transition.

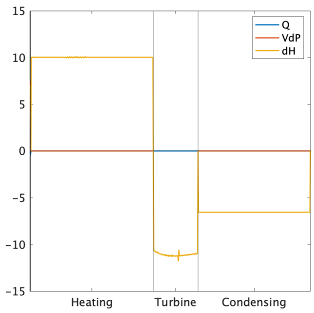

Figure 4. How the enthalpy, H, changes during the different stages of the steam engine cycle for the nuclear power plant.

This figure shows that, during the heating stage of the cycle, all of the change in internal energy of the water is due to the heat being added to the system, regardless if the water is in a liquid or gaseous form. During the turbine stage of the cycle, the figure shows that all of the change in energy of the system is responsible for the work created by the turbine.

To analyze how the internal energy, enthalpy, and Gibbs free energy change in tandem, the following plot is produced:

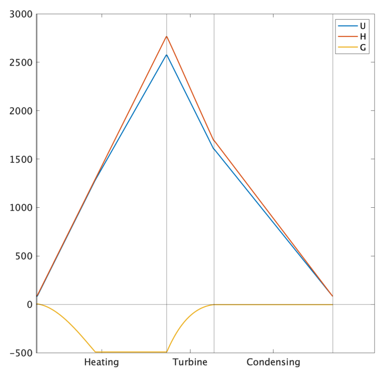

Figure 5. How U, H, and G all change with respect to each other for the nuclear power plant.

The line for dG appears the way it does in Fig. (5) because, during the isobaric heating stage, $dG = -SdT$, and $dT = 0$ at the phase boundary since, again, when more energy is added to the water at this stage, it is either used to make more of the liquid water become gaseous or to raise the volume of the container that encloses the gas. So before the purely liquid water becomes a mixture of both liquid and gas, the Gibbs free energy has a negative differential value determined by the entropy and change in temperature of the system.

When the water hits the phase boundary, it becomes both a liquid and gas, meaning that any additional heat that's added will either contribute to the expansion in volume of the gas or contribute to the amount of liquid water that has become gaseous. This is why the slope of H's line remains constant during the heating phase while U has a decrease in slope. Since

$$dU = TdS - PdV + \mu dN \tag{3}$$

and

$$dH = dU + d(PV) \tag{4}$$

where $dN = 0$ since the number of particles does not change during any step of the Rankine cycle and $dP = 0$ due to the heating being an isobaric process and since any energy added to the water after the phase boundary either causes the volume of the gas to expand or more of the liquid water to become gaseous, the PdV term in the definition of dU will begin to grow as more liquid water undergoes a phase transition into gaseous state, decreasing the slope of U's line after the phase transition occurs.

It should be noted that during each phase of the power plant, either U or H is perfectly linear in Fig. (5) because of Eq. (**??**) and because $dH = TdS + VdP$. The water is pumped isothermally without changing the volume of the container, meaning that dU and dH will be linear. The isobaric ($dP = 0$) heating stage will also result in a linear line for dH and dU that will contain a discontinuity in its line at the phase transition, for reasons explained earlier. The isentropic turbine stage features linear lines because $dS = 0$ and the final condensing stage features linear lines because both the volume and temperature of the liquid decrease during this stage. The Gibbs free energy is constant during the portion of the heating phase after the phase transition occurs because $dP = dN = 0$ and temperature is linearly increased. The Gibbs free energy is constant at zero during the condensing phase because $dN = dT = dP = 0$ during this phase.

This analysis of power plants enforces a thorough understanding of the various forms of energies and why different forms must be considered in different scenarios to reveal new behavior of a system. Fig. (5) forced me to think about what really happens during a phase transition between liquid and gas: I did not have a good mental picture for this transition and where the input energy went during it. I have now learned that when energy is added to water at the phase boundary between liquid and gas, the energy can either contribute to more of the liquid becoming a gas or can either be given directly to the gas, causing the volume of whatever container hold this mixture of liquid and gas to increase.

IV. CONCLUSIONS

In this study, the Rankine cycle as used in modern power plants was analyzed to reveal the underlying thermodynamics and to provide a strong framework for how these power plants operate. For future investigation, I would like to study the Helmholtz energy further and get a better understanding for why it was not necessary to analyze in this study to arrive at results of this study. I would also like to carry out the analysis detailed in this paper for the supercritical power plant since it was found that this was the most efficiency power plant out of the three considered. I would like to understand what thermodynamic properties the supercritical power plant exploits to increase efficiency and would like to consider any potential negatives to its use. It could be the case that the supercritical power plant is the most ideal but is very impractical in terms of its construction.

From this paper's analysis, I want to believe that increasing the maximum volume that the gas phase of the water is allowed to expand to would increase the efficiency of these power plants. Changing how quickly heat is added to the water to cause the phase transition would not change the efficiency since the same amount of heat would still be used. To increase efficiency of the engine, you would need to decrease the amount of heat needed to operate the cycle or increase its work produced and, since this work is defined as $W = -PdV$, it makes sense that allowing the volume to have a greater expansion would increase the work of the cycle. I would also assume that this would require additional energy to allow the gaseous water to expand to greater volumes, though. It should be noted that an extension upon the traditional Rankine cycle, called the reheating Rankine cycle is a current method used to increase the cycle's efficiency.

REFERENCES

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[4] *Steam Tables (.xslx).* URL: https://learncheme.com/student-resources/steam-tables/.