3+1 Decomposition of Scalar-Vector-Tensor Gravity

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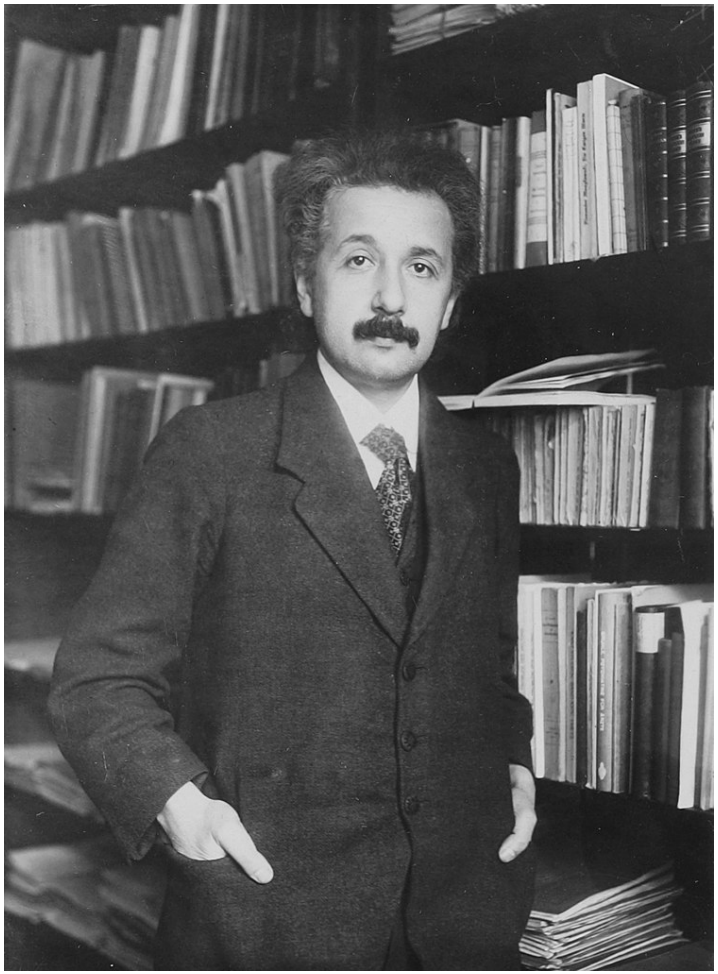
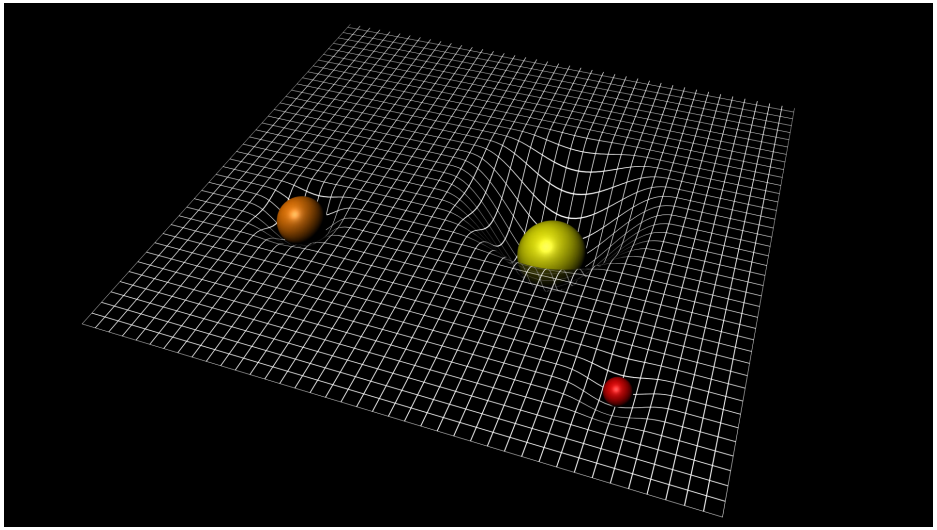
General Relativity





Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Problems with GR:

- Singularities ∴ QG?

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)^2 c^2\,dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2\,d\Omega^2 \qquad (1)$$

- Dark matter and dark energy ∴ *modify* general relativity?

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More theories of gravity $\implies$ more field equations to be analyzed

Modified Gravity

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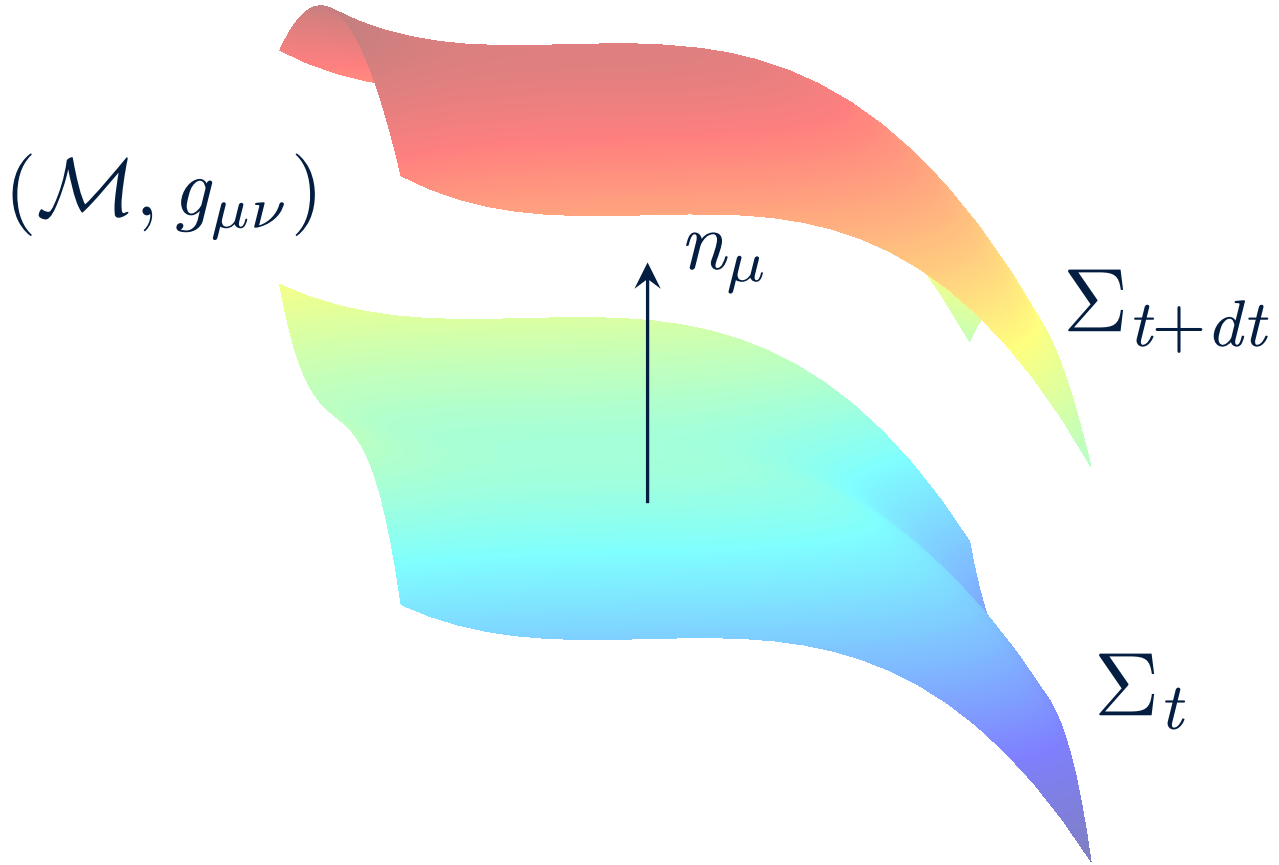
Scalar-Vector-Tensor gravity:

- Add scalar field ϕ (fixes DE) and vector field A_μ (fixes DM)

$$g_{\mu\nu} \to \phi, A_\mu, g_{\mu\nu} \qquad (2)$$

3+1 Decomposition

4D manifold $\mathcal{M} \rightarrow$ 3D hypersurfaces Σ_t evolved over time $(+1)$



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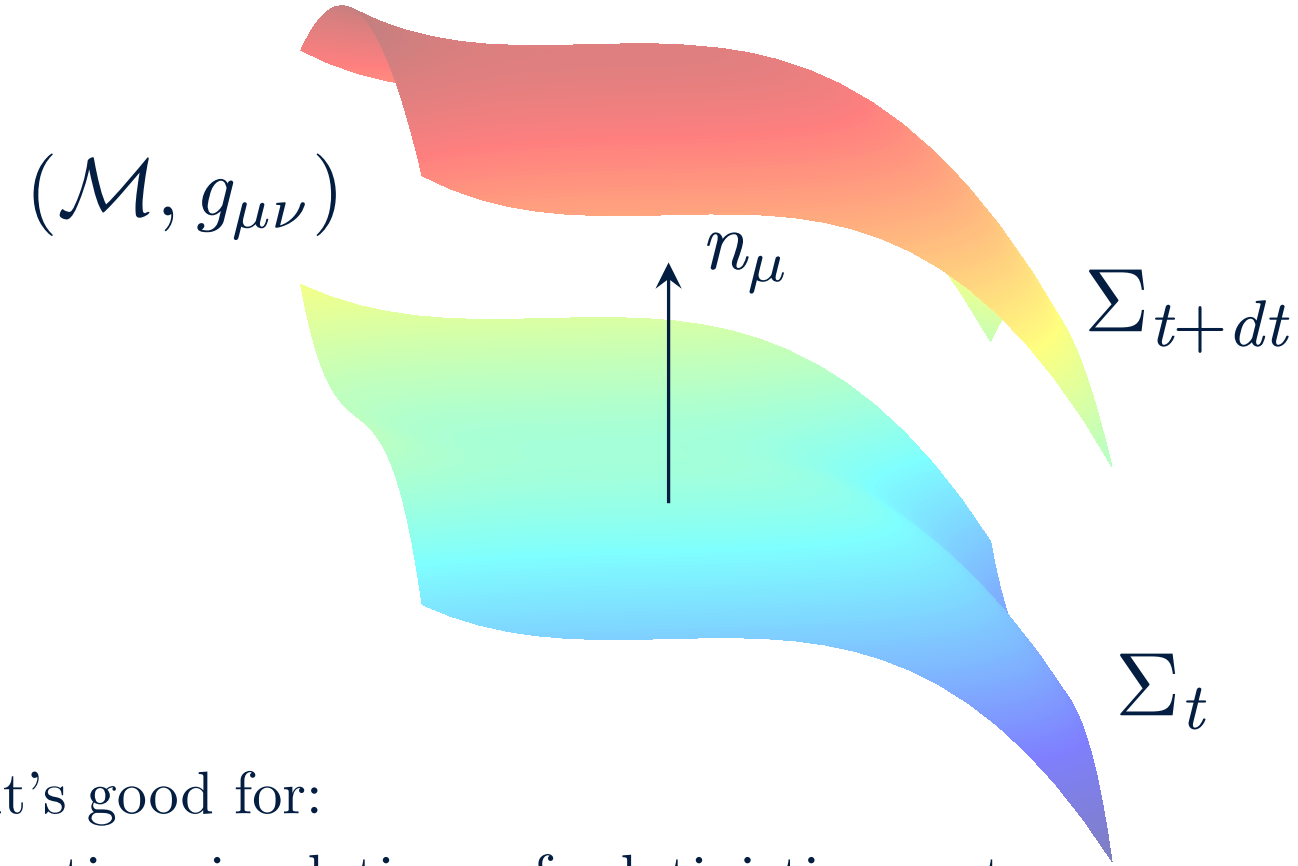

What it's good for:

- Creating simulations of relativistic events
- Specifying initial conditions
- Quantizing gravitational field (LQG ∼1980s)
- **Math theses**

Conclusion

- I am applying **3+1 decomposition** to a modified theory of gravity (called **Scalar-Vector-Tensor gravity**)
- I see this as a purely mathematical problem:
  I do believe dark energy and dark matter exist

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"Since the mathematicians have invaded the theory of relativity,
I do not understand it myself anymore"
- Albert Einstein