Modeling Binary Compact Object Merger Events Detected by the LIGO and Virgo Gravitational Wave Observatories

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Abstract

In this investigation, we model multiple neutron star and black hole merger events detected in the LIGO-Virgo Collaboration. We use Kepler's Laws and Newtonian mechanics to model an infalling system of two objects with equal masses. We predict the expected increase in frequency or "chirp" of the infalling binary and compare that to what is found in the LIGO-Virgo database (presented in an adjacent poster). In the initial period during which the Newtonian approximation is valid, we find reasonable agreement between our model and the results from the LIGO-Virgo Collaboration, thus verifying the basic physics of the infall. We also estimate the amount of gravitational wave energy emitted during the entire process. This provides a better understanding of the nature of these merger events and why gravitational waves are emitted by these merging compact objects.

Introduction and Background

- Gravitational waves were first predicted by Albert Einstein in 1916
- These ripples in spacetime are caused by merging binary compact object systems (black hole and neutron star pairs)
- Gravitational waves were first detected by the Laser Interferometer Gravitational Observatory (LIGO) and Virgo Observatory instruments
- Strain data from the instruments helps scientists understand the properties of the system and model the event

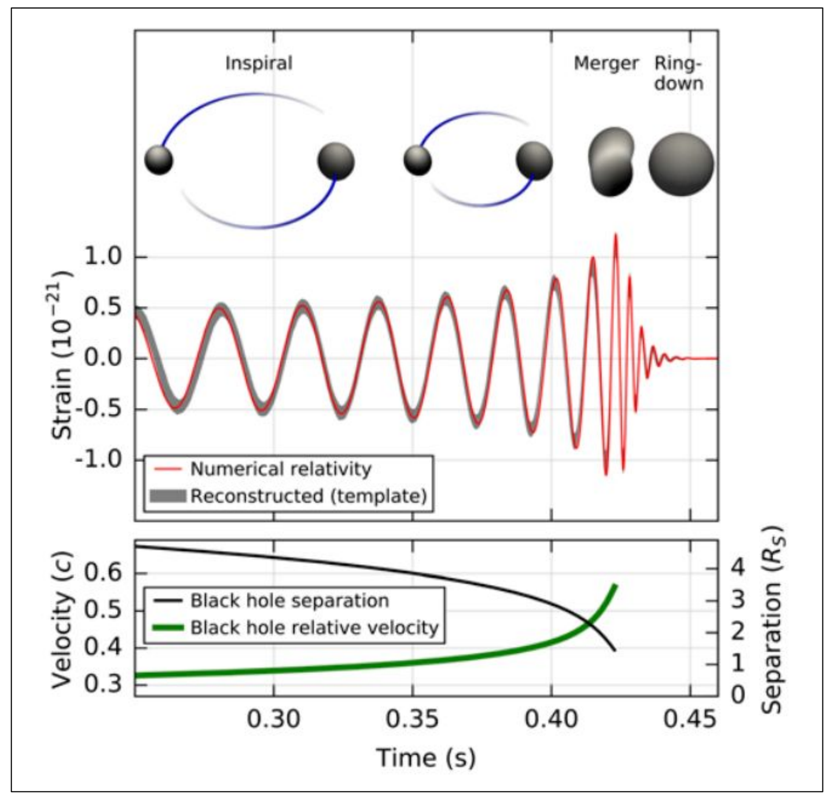
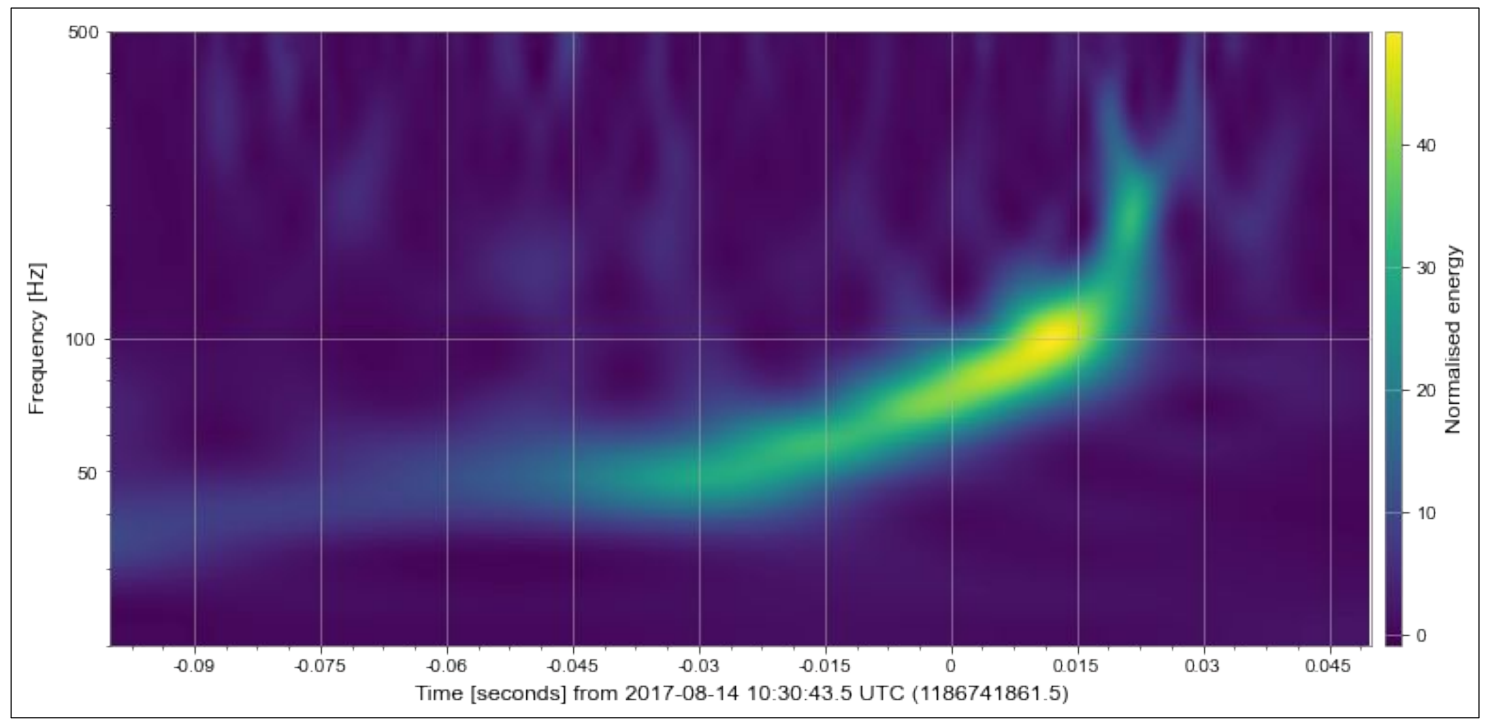


Figure 1: This image shows a visual representation of an infalling black hole pair, the instrument's strain data, and the behavior of the black hole separation and velocity.

- The frequency of the revolutions increases rapidly as the orbit decays
- This results in a "chirp" pattern in the frequency, shown below
- This plot shows a sample spectrogram obtained in companion poster analysis



The Quadrupole Moment

- The system of compact masses is shown in Figure 2
- The behavior of the system's angular momentum is captured by the quadrupole moment tensor, shown below
- The system can also be depicted as a single mass (reduced mass *μ*) orbiting a second stationary mass (combined mass *M*) at separation $r = r_1 + r_2$
- This will be used to simplify the expression for quadrupole moment

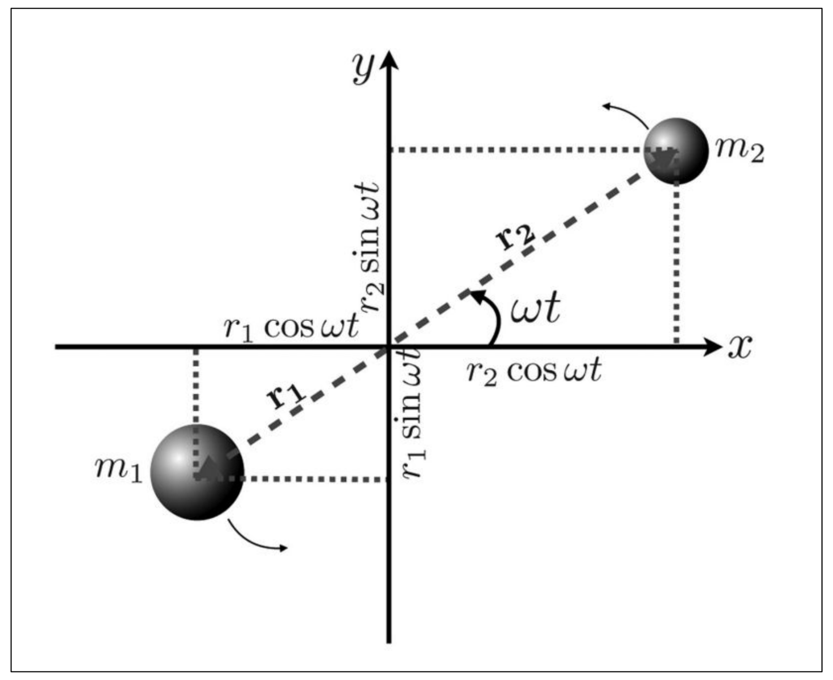


Figure 2: The two body system used in our model. Distances are represented in polar coordinates.

$$Q^A_{ij} = \begin{pmatrix} \frac{2}{3}x_A^2 - \frac{1}{3}y_A^2 & x_A y_A & 0 \\ x_A y_A & \frac{2}{3}y_A^2 - \frac{1}{3}x_A^2 & 0 \\ 0 & 0 & -\frac{1}{3}r_A^2 \end{pmatrix} = \frac{m_A r_A^2}{2} I_{ij} = \frac{\mu r^2}{2} \begin{pmatrix} cos(\omega t) + \frac{1}{3} & sin(2\omega t) & 0 \\ sin(2\omega t & \frac{1}{3} - cos(2\omega t) & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$$

Equation 1: The quadrupole moment matrix simplified and written in terms of reduced mass and separation

Newtonian Approximation for GW Emission

- Einstein's quadrupole formula (below) yields the energy lost as gravitational waves per unit of time

$$\frac{d}{dt}E_{GW} = \frac{1}{5}\frac{G}{c^5}\sum_{i,j=1}^{3}\left(\frac{d^3Q_{ij}}{dt^3}\right)^2 = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6$$

Equation 2: Einstein's quadrupole formula. By adding together the squared third derivatives of each of the quadrupole moment matrix elements, an expression for gravitational wave power can be derived.

- The energy radiated away by gravitational waves is equal in magnitude to the change in orbital energy of the system
- Using this fact and Kepler's Third Law, an expression for the change in angular momentum can be derived
- The expression is written in terms of chirp mass, a single quantity determined by the masses of the two orbiting bodies

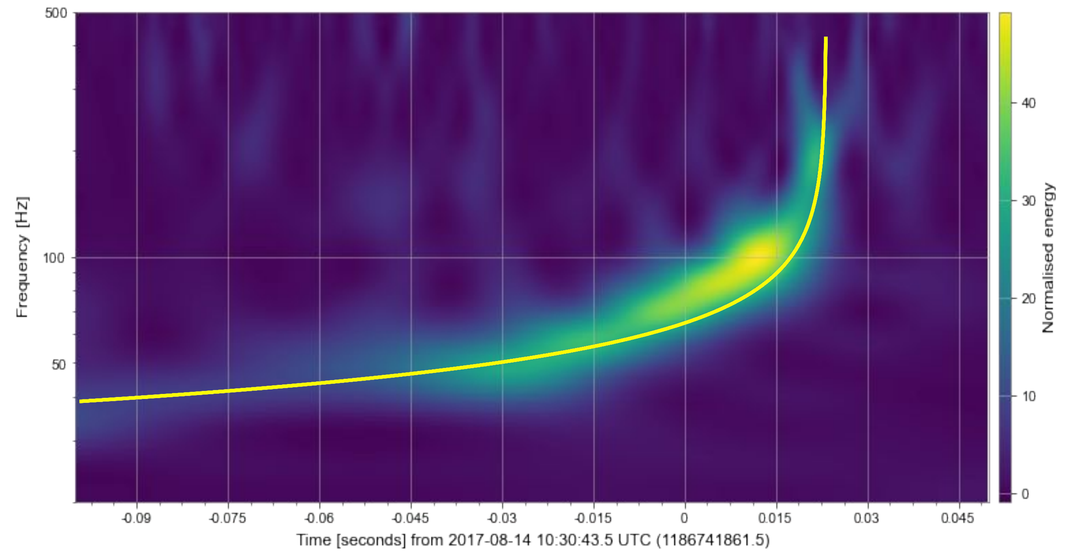
$$\dot{\omega}^3 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}}(G\mathcal{M})^5 \qquad \mathcal{M} = (\mu^3 M^2)^{\frac{1}{5}}$$

Equation 3: This is the differential equation relating the change in angular momentum of the system to various parameters. Kepler's Third Law can be used again to determine the change in separation as well. The other parameters included here are the instantaneous angular momentum and the chirp mass of the system.

- This differential equation will be the basis of our Python model of the system of objects

Python Modelling

- Can roughly model compact object infall by using first-order Newtonian approximation
- As objects get closer together, Newtonian approximation breaks down and general relativity becomes necessary. Using GR is beyond the scope of this research
- Using an initial angular frequency and initial masses, the program finds the radius and change in angular frequency, and then recursively runs until Newtonian approximation fails
- The program returns plots of frequency versus time, which can be compared to LIGO's official data



- The yellow/green line is LIGO's data, the plain yellow line was generated by our program
- Approximation isn't exact, but it is close, as the general shape is the same
- The program can also be used to model the radius versus time, change in frequency versus time, and the energy dissipated as gravitational waves during the merger

Conclusion

- Gravitational waves can be modelled to a reasonable degree of accuracy using a first order Newtonian approximation
- This approximation is derived from the quadrupole moment matrix and Kepler's Third Law
- A Newtonian model based on a number of parameters can be developed using Python
- This model can be tested with real strain data from LIGO
- Plots of orbital frequency as a function of time can be obtained, displaying the characteristic "chirp" pattern of a compact object merger event
- The differential equation and LIGO data based model come together to accurately represent the behavior of a merger event, showing that both the theoretical and observational assessments of the system are accurate.

Acknowledgements

- Dr. Jean Quashnock, for overseeing our research
- LIGO, for releasing publicly-accessible data
- Our companion poster, hosted by Andrew Valentini and Chance Hoskinson

References:

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