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Measuring Quasinormal Modes of Simulated Binary Black Hole Mergers in the SXS Catalog

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Abstract

Black hole spectroscopy is the prospect of using gravitational wave emissions as a superposition of damped sinusoids, called quasinormal modes (QNMs), to determine the properties of the remnant black hole. Building on recent work by researchers in the field [1][2], we test the prospects of detectability of the QNMs of binary black hole (BBH) merger events and analyze the dependence of the first overtone on the remnant's mass and spin. Using numerical relativity simulations from the Simulating eXtreme Spacetimes (SXS) project, we confirm for SXS:BBH:0305 that the (2,2,1) overtone features an amplitude comparable to that of the (2,2,0) fundamental mode. We ultimately aim to develop tools that utilize LVK data to test whether the detected QNMs are consistent with our current understanding of the Kerr geometry and general relativity.

Introduction and Background

- Gravitational waves were first predicted by Albert Einstein in 1916
- These ripples in spacetime are caused by merging binary compact object systems (black hole and neutron star pairs)
- Gravitational waves were first detected by the Laser Interferometer Gravitational Observatory (LIGO) and Virgo Observatory instruments
- Strain data from the instruments helps scientists understand the properties of the system and model the event

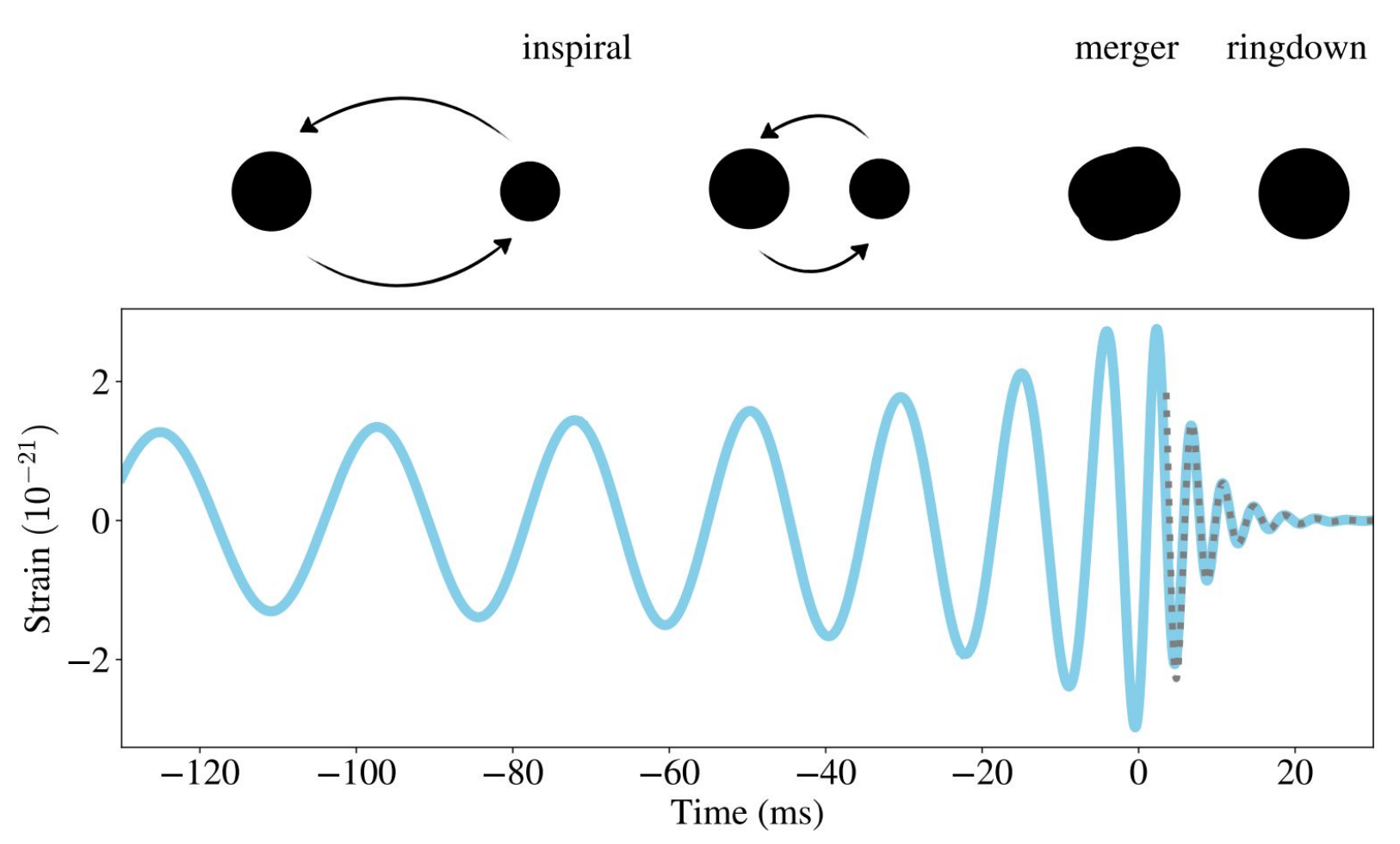

Figure 1: This plot shows the typical evolution of a binary system as the three phases that a binary merger experiences, and a waveform that follows with each phase. A dotted curve is placed over the last section of the waveform to highlight the ringdown phase. [1]

Ringdown

- Gravitational waves are emitted in 3 phases; characterized by the *inspiral*, which can be described using Post-Newtonian approximations, the highly chaotic *merger*, described using Numerical Relativity (NR) simulations, and the linear *ringdown* that can be expressed as a sum of damped sinusoids, known as quasinormal modes.
- The oscillation and damping time of each quasinormal mode should be uniquely determined by the mass and spin properties of the final black hole, which is relaxing to an unperturbed state in the ringdown.
- Multi-mode analysis of the QNMs would allow the no-hair theorem and general relativity to be tested, which emphasizes the importance of detecting more than one overtone in a signal.

Importance of Overtones

- The fundamental mode alone is not enough to recover the true source parameters (mass and spin)
- Ref. [3] proposes that the inclusion of up to 7 overtones provides an accurate representation of the waveform for any ringdown:

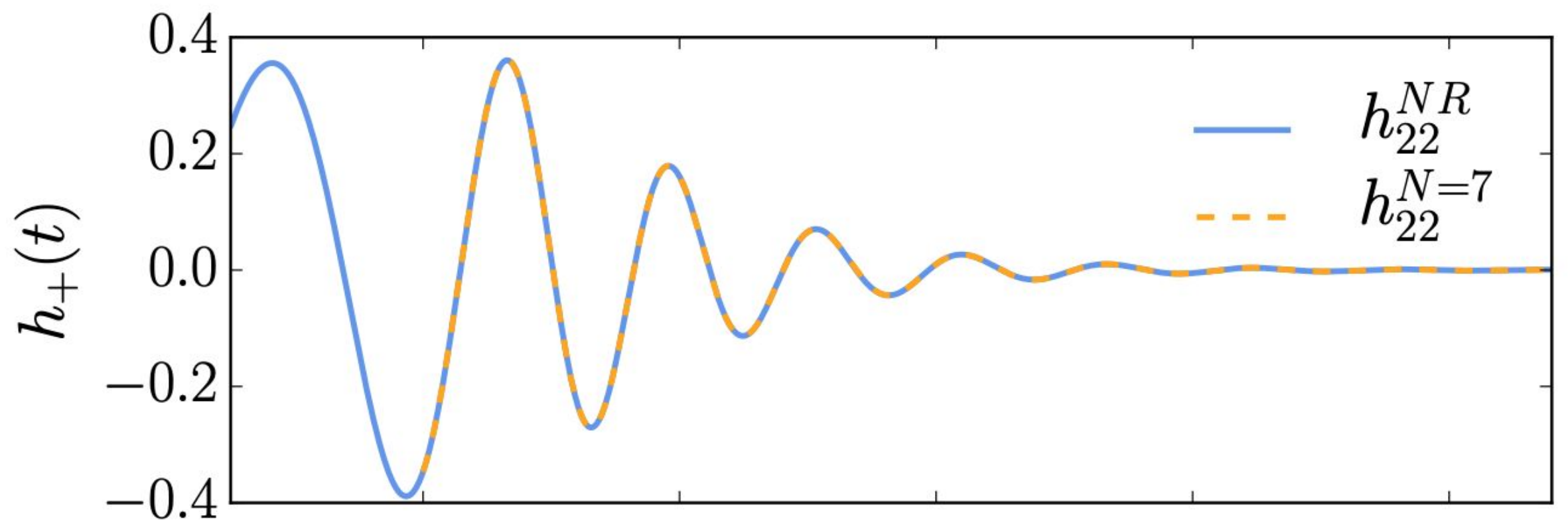

Figure 2: This figure shows a waveform modelled by the addition of 7 overtones over the SXS simulation for SXS:0305 [3]

- The significance of a fit with many overtones is debated. Our study will focus on measuring the fundamental and its first overtone.

Fundamental + First Overtone

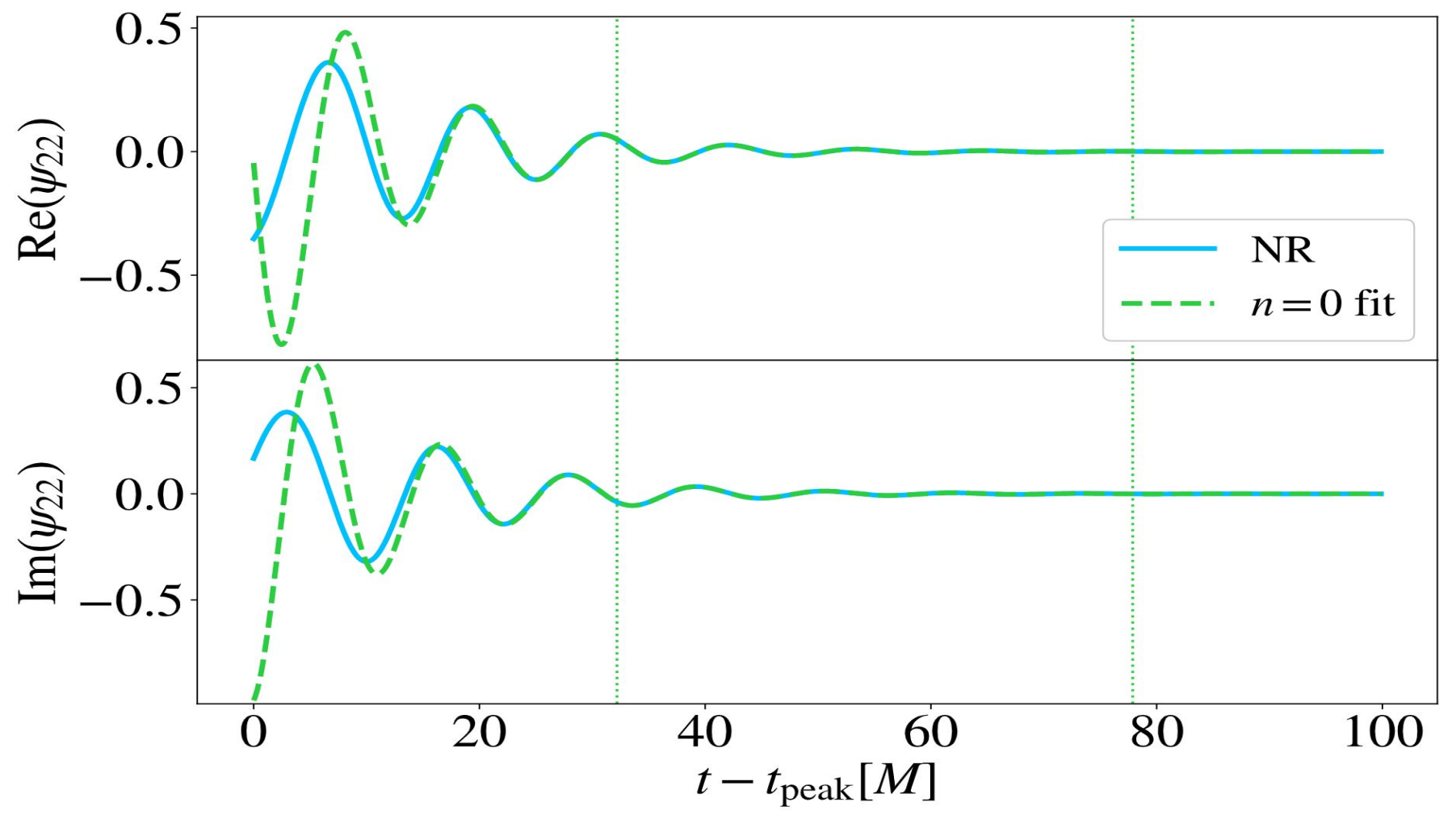

Figure 3: This figure shows the real and imaginary components of the ringdown waveform for the SXS:0305 simulation. The early discontinuity shows that the fundamental mode alone is not enough to determine the true source parameters of the merger

- To determine the initial time of the ringdown period, we perform a non-linear fit of the form given in the following equation looking for the first minimum of the mismatch between the NR data and it [1]

$$\psi_{\ell mn} = A_{\ell mn} e^{-\omega^i_{\ell mn} t}[cos(\omega^r_{\ell mn} t - \phi_{\ell mn}) + isin(\omega^r_{\ell mn} t - \phi_{\ell mn})]$$

- This waveform can be expanded as a sum of QNMs to consider the contributions of overtones:

$$\sum_n \psi_{\ell mn}$$

SXS Merger Event Data

- Simulating eXtreme Spacetimes (SXS) provides a catalog of simulated merger events. [4]
- Our investigation focuses on three primary groups of simulations from the SXS catalog:
    - Events with a high initial mass ratio
    - Even with a high effective spin value
    - Events with mass ratios and effective spins similar to those of SXS:0305 simulation (modelled after the first confident detection GW150914)

Results

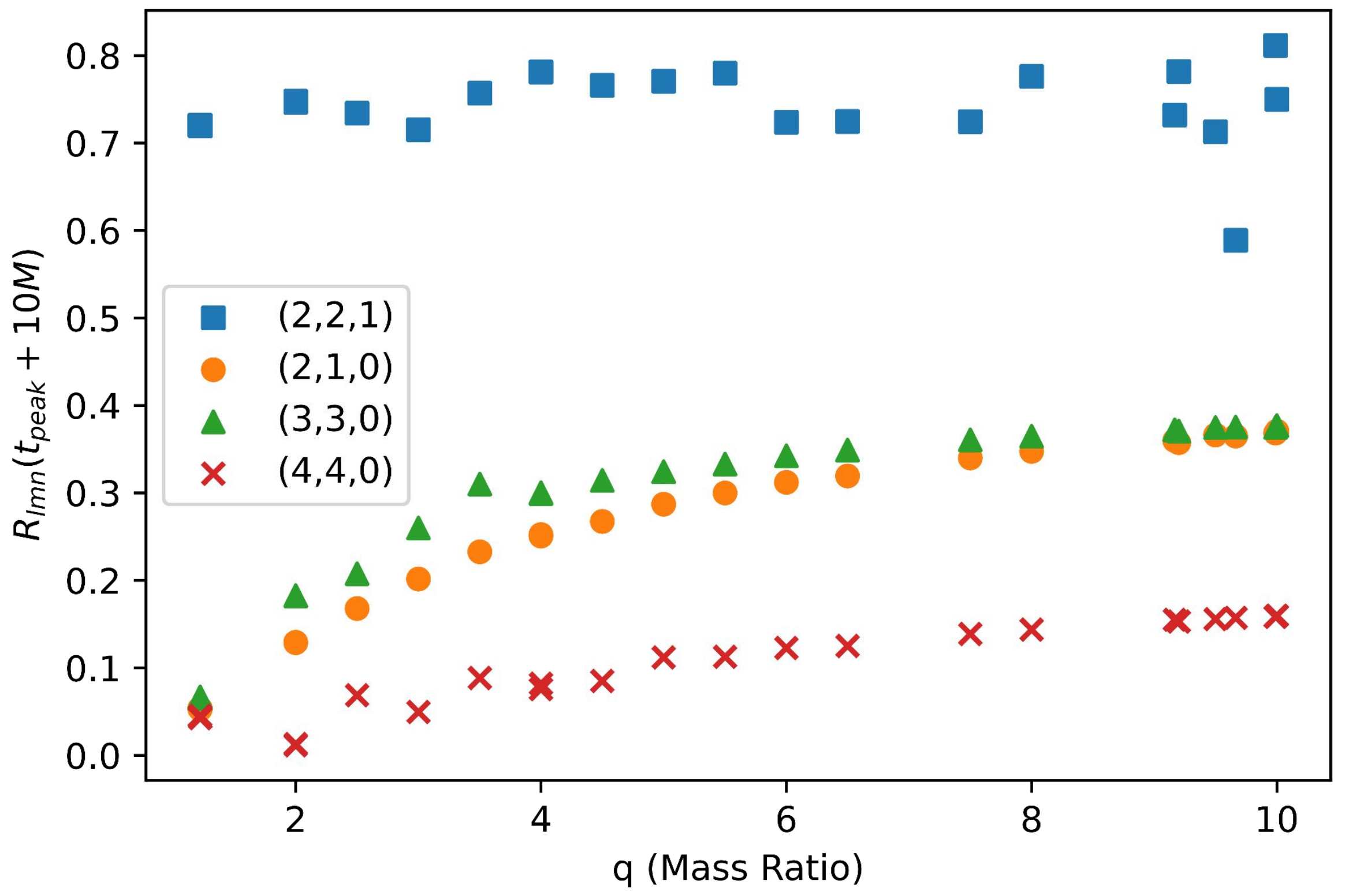

Figure 4: Amplitude ratio between the fundamental mode and the higher harmonics (l,m,n) = (2,2,1), (2,1,0), (3,3,0), (4,4,0). Both the fundamental and higher harmonics are evaluated at t = tpeak + 10M as a function of q (mass ratio). The first overtone clearly has a larger amplitude ratio than all the fundamentals for q < 5. For q between 5 and 10, (2,1,0) and (3,3,0) have comparable ratios.

Tables 1 and **2** confirm the theoretical prediction that the amplitude of a gravitational wave is inversely proportional to its remnant mass

q = 1.218 (SXS:1353)

(ℓ, m, n)	$A_{\ell mn}$	$\phi_{\ell mn}$	$\omega^r_{\ell mn} MG/c^3$	$\omega^i_{\ell mn} MG/c^3$
$(2,2,0)$	0.414	6.251	0.555	0.085
$(2,2,1)$	0.301	3.296	0.543	0.257
$(3,3,0)$	0.027	5.788	0.881	0.087
$(4,4,0)$	0.018	4.584	1.193	0.088
$(2,1,0)$	0.022	4.995	0.476	0.086

q = 9.989 (SXS:0185)

(ℓ, m, n)	$A_{\ell mn}$	$\phi_{\ell mn}$	$\omega^r_{\ell mn} MG/c^3$	$\omega^i_{\ell mn} MG/c^3$
$(2,2,0)$	0.137	3.826	0.415	0.088
$(2,2,1)$	0.354	7.692	0.393	0.271
$(3,3,0)$	0.051	0.572	0.666	0.092
$(4,4,0)$	0.021	3.847	0.901	0.093
$(2,1,0)$	0.051	2.501	0.396	0.088

Conclusions

- We confirm that the (2,2,1) overtone features amplitude comparable to the fundamental for SXS:BBH:0305
- For mass ratios between 1 and 5, the (2,2,1) has the greatest amplitude, and between 5 and 10 the amplitude is comparable for the higher harmonics of (2,1,0) and (3,3,0)
- We hope to reproduce similar analyses with the consideration of higher overtones.

Acknowledgements

- Iara Ota, for assisting our research and releasing publicly-accessible code
- LIGO and SXS, for releasing publicly-accessible data
- Dr. Jean Quashnock, for overseeing our research

References:

[1] Ota, Iara. "Black hole spectroscopy: prospects for testing the nature of black holes with gravitational wave observations." *arXiv preprint arXiv:2208.07980* (2022).
[2] Ota, Iara, and Cecilia Chirenti. "Overtones or higher harmonics? Prospects for testing the no-hair theorem with gravitational wave detections." *Physical Review D* 101.10 (2020): 104005.
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[4] SXS Gravitational Waveform Database. Available online: https://data.black-holes.org/waveforms/index.html