Portfolio Comparison under Different Volatility Measurements

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Phase 1: Simulation

Portfolio

We build a portfolio of options with the following characteristics:

- It contains calls and puts of different stocks
- The expiration days of the options are different
- The strike price of the options is different
- The moneyness differs: some contracts are in-the-money, some at-the-money, and some out-of-the-money

Volatility Models

Given the underlying stocks, we model the volatility with four different methods:

1. **Black-Scholes:** Historical volatility, i.e., historical standard deviation of the log-returns r_t.
2. **Exponentially Weighted Moving Average (EWMA) Volatility:** We assume the volatility evolves according to the model

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2$$

   where $\lambda = 0.94$ (this is the usual value taken in finance.)

Volatility Models

Given the underlying stocks, we model the volatility with four different methods:

4. **GARCH(1,1) model:** We assume the volatility evolves according to the model

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

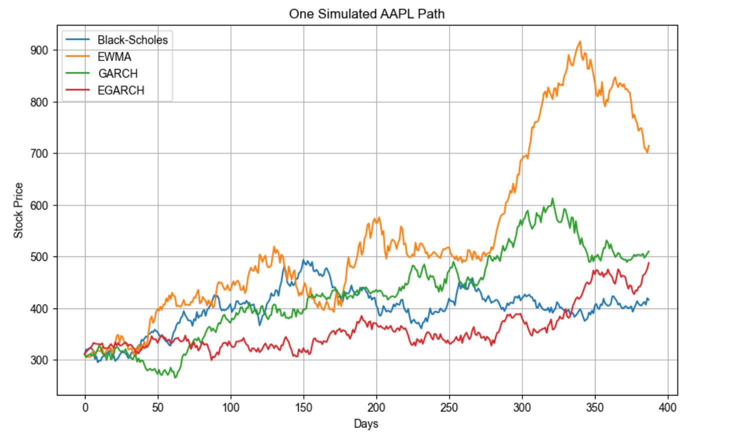
where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$ are parameters to fit.

5. **EGARCH(1,1) model:** We assume the volatility evolves according to the model

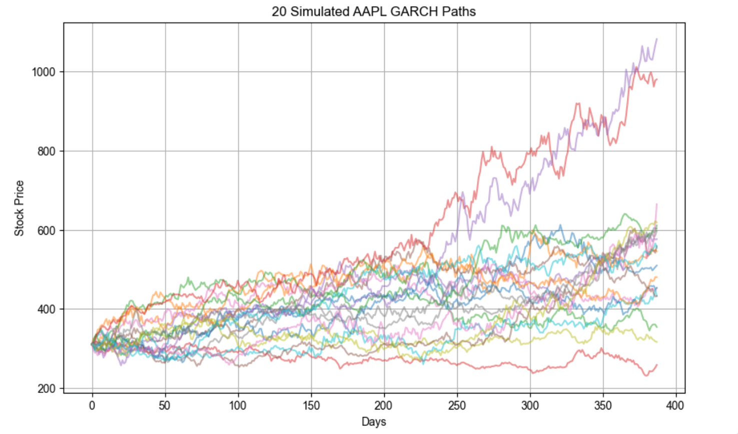
$$\log(\sigma_t^2) = \omega + \alpha(|Z_{t-1}| - \mathbb{E}|Z_{t-1}|) + \gamma Z_{t-1} + \beta \log(\sigma_{t-1}^2),$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and γ are parameters to fit.

Volatility Simulation Example



Stock Simulation Example



Phase 2: Greeks Computation

Delta Hedging Review

Compute return for each day when held options have changed dV while we short Δ shares:

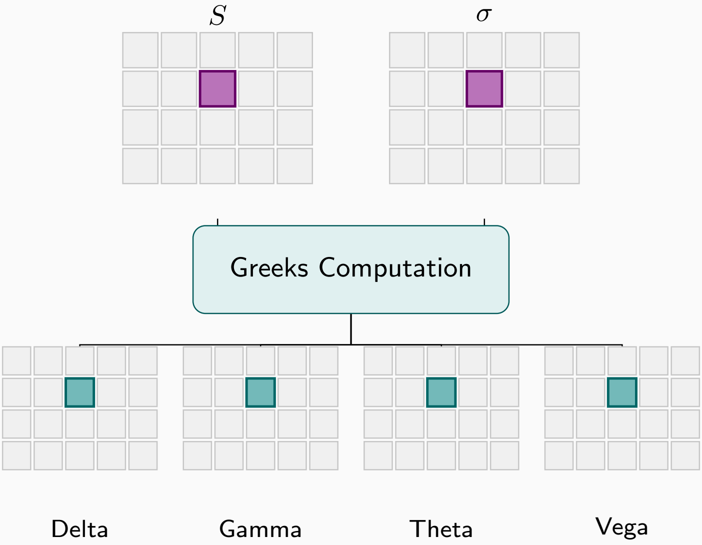
$$\text{P\&L} = dV - \Delta dS$$

where $V(S, \sigma, t)$, so we must compute

$$\Delta = \frac{\partial V}{\partial S}, \quad \Gamma = \frac{\partial^2 V}{\partial S^2}, \quad \Theta = \frac{\partial V}{\partial t}, \;\; \& \;\; \text{vega} = \frac{\partial V}{\partial \sigma}$$

to determine daily P&L

Greeks Computation



rows = paths, columns = days

Phase 3: Profit & Loss Evaluation

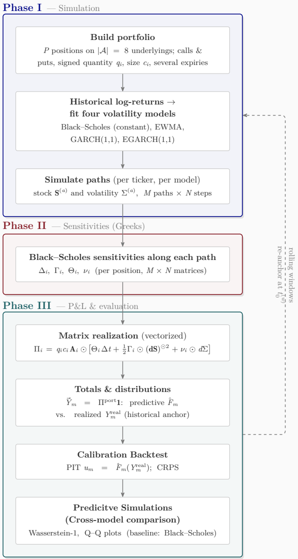
Model Evaluation Strategy

One-day P&L of the delta-hedged short position

For a written option, over one day the hedged position changes by:

$$\Delta\Pi_{k+1} = \underbrace{-\,\theta_k\,\Delta t}_{\text{premium "rent" earned }(>0)} \;\underbrace{-\,\tfrac{1}{2}\,\Gamma_k\,(\Delta S_{k+1})^2}_{\text{damage from the stock moving }(\le 0)} \;\underbrace{-\,\nu_k\,\Delta\sigma_{k+1}}_{\text{loss if implied vol rises}}$$

A seller profits when the stock stays calmer than the option's implied volatility

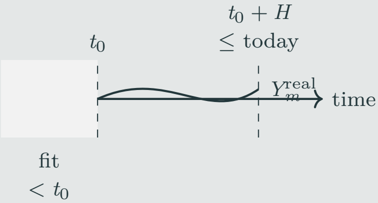


Backtest vs. Predict

Evaulating how each model's predicative distribution measures against the past

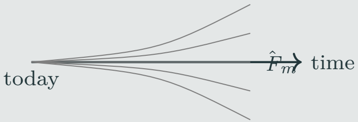
Calibration Backtest

For each past anchor t_0 with an elapsed horizon, we fit on pre-t_0 returns and hedge the realized path into one outcome Y_m^{real}; rolling many anchors scores each model's calibration.



Predictive Simulation

Starting from today, we run each model's M simulated paths forward and hedge every one, collecting the results into a full PL distribution $\hat{F}_m$ using no historical data.



Experiment Setup

Portfolio 17 positions, 8 underlyings (AAPL, META, AMZN, MSFT, NVDA, KO, GE, MCD); calls/puts, long/short, 0.90–1.10× spot.

Models BS, EWMA(λ=0.94), GARCH(1,1), EGARCH(1,1); returns since 2021, 252-day clock.

Predict 1000 paths/underlying forward from today $\to \hat{F}_m$.

Backtest 120-day windows, no overlap, 500 paths, no look-ahead $\to Y_m^{\text{real}}$.

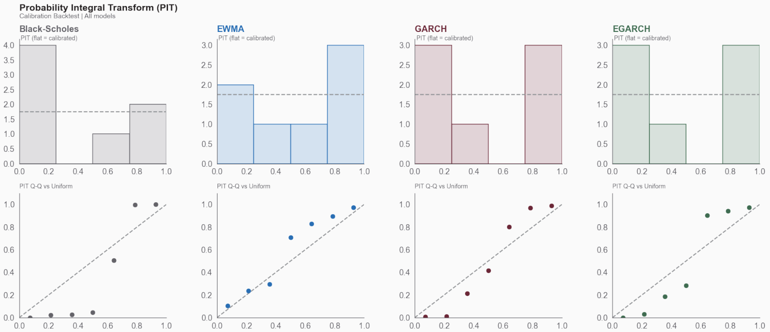
Calibration Backtest: Probability Integral Transform

Understanding which quantile did the historical data land in

$$u_m = \hat{F}_m\big(Y_m^{\text{real}}\big) \in [0,1], \qquad \hat{F}_m(y) = \frac{1}{M}\sum_{j=1}^{M} \mathbf{1}\{\, Y_m^{(j)} \le y\}$$

where

- $\hat{F}_m$: Predictive CDF
- Y_m^{real}, $Y_m^{(j)}$: the realized P&L vs. the j-th simulated P&L



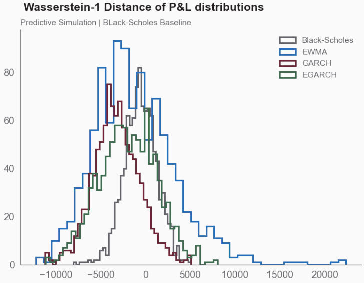
Predictive Simulation: Wasserstein-1 Distance

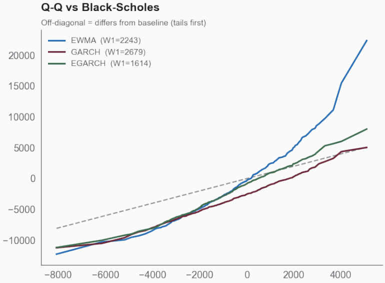
How far does each model's P&L distribution sit from Black–Scholes?

$$W_1(m) = \frac{1}{M} \sum_{j=1}^{M} \left| \, Y_{m,(j)} - Y_{\mathrm{BS},(j)} \, \right|$$

where

- $W_1(m)$: mean gap between model m's outcomes and Black–Scholes'
- $Y_{m,(j)}$: the j-th *sorted* simulated P&L under model m
- $Y_{\mathrm{BS},(j)}$: the j-th sorted simulated P&L under the Black–Scholes baseline
- M: number of simulated paths





Discussion

- GARCH seems to change the forceast and calibrates to reality better than the other models
- BS seemed to perform the worst in the calibration backtest
- All models seem to depart from BS (left-tail departures)
- Future work will be to includes to analyze how these results would differ conditioned on moneyness, expires etc.

References

- Cumby, Robert, Stephen Figlewski, and Joel Hasbrouck (1993). **"Forecasting volatility and correlations with EGARCH models".** In: *Journal of Derivatives* 1.2, pp. 51–63.
- Hull, John (2022). ***Options, Futures, and Other Derivatives, 11.***
- Wilmott, Paul, Sam Howison, and Jeff Dewynne (1995). ***The mathematics of financial derivatives: a student introduction.*** Cambridge university press.